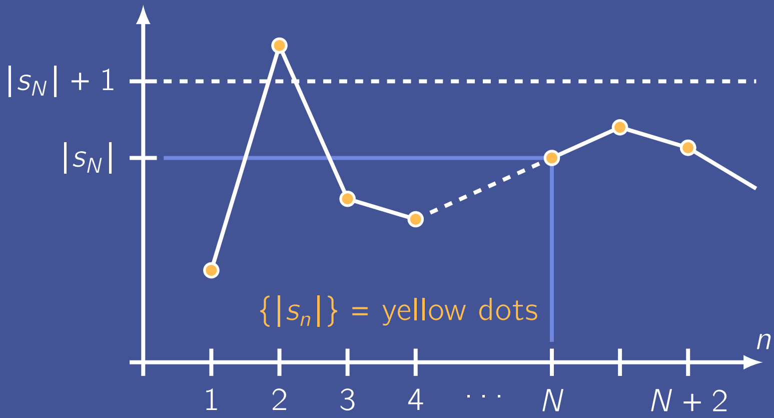
Lemma: Cauchy sequences are bounded.



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Proof

Let $\{s_n\}$ be a Cauchy sequence. It suffices to show there is $M > 0$ such that $|s_n| \leq M$ for all n.

Since $\{s_n\}$ is Cauchy, there is N such that

$$|s_n - s_m| \leq 1, \quad \text{for all } m, n \geq N.$$

Picking $m = N$ and using the triangle inequality reveals

$$|s_n| \leq |s_N| + |s_N - s_n| \leq |s_N| + 1, \quad \text{for all } n \geq N.$$

This implies

$$|s_n| \leq \underbrace{\max\{|s_1|, |s_2|, \ldots, |s_{N-1}|, |s_N| + 1\}}_{\text{set } M \text{ to be this expression}}, \quad \text{for all } n,$$

and the proof is complete. ■

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